

Graphical Representation of an Accounting Balance

Amaury S. Amaral

Pontifical Catholic University of São Paulo/Accounting Department

Victor E. Camargo

University of São Paulo/Interdisciplinary Nucleus of Complex Systems

Fernando F. Ferreira

University of São Paulo/Interdisciplinary Nucleus of Complex Systems

ABSTRACT: Our proposal is a theoretical essay with the objective of disposing account flow data in a possible network topology where we could evaluate the clusters eventually occurred and thus try to identify a possible trend in the geometric arrangement obtained by a topology intrinsic to the company¹. To do this, we use graph theories associated to the balances of the G/L accounts in different periods. We also use an algorithm called Kamada-Kawai to dispose of such a topology that uses the disposition of forces and potential energy well known in physics. It was possible to arrange the data in a two-dimensional way so that the edges have equal lengths and in concomitance the smallest possible number of cross edges of this data. By the mechanism of the algorithm a system of potential forces is assigned to the edges, analogously to a system of springs also known in Physics. In this way the system can be stabilized by minimizing the energy assigned to the system. It is possible to see the data of a trial balance arranged in the form of graphs where the flows of the accounts could be observed. Furthermore, one can notice the tendency of the geometrical arrangement of the accounts that the model can present.

KEY WORD: accounting balance; graphs; accounting network; Kamada-Kawai algorithm

Date of Submission: 05-01-2021

Date of Acceptance: 20-01-2021

I. INTRODUCTION

Reading models of implemented business into balances are performed a long time ago through static models of data arranged in tabular shape into an ordered and logic sequence. Data are dynamics and move through the accounts in an ordered pair (α , β) also known as double entries. So, to each α there is a corresponding β .

Due to the evolution of processes brought by the IFRS to the dynamics of current balances there is a big interest in knowing the evolution of these data in timeline especially during economical management.

A negative financial situation can be harmful to current economic situation, and if such occurs it could be due to opportunity problems faced by the entity.

The static analysis of an accounting balance will be interesting in situations where future operations may occur and such study will be obtained through sensibility analysis in better estimates.

Diagnostics occur after the reduction of classifications and grouping of daily movements in an accumulated and synthetized expression in the period of interest. Thus the performance display of business operations occurs at the end of the period, represented in the balance sheet, that is, at the year-end closing whose date is previously defined or agreed.

Each part of this balance sheet or each item is yet a part of company equity defined by the expression (Net Equity = Asset - Liabilities).

The method is so ancient as the own accounting history. It goes back to era in which small farmers or small animal breeders compared their goods to a set of small stones assigning to them a "value" that would represent their stocks. This assignment of "value" evolved until our famous decoder of existing practices, Franciscan Friar Pacioli, described it as an ordered pair whose sum totalize the liabilities on the left and the credits on the right. He assured that if the sums at the time of conclusion do not be equal so an error has occurred: "... it will be better to find the error with the intelligence given by God and with the acquired reasoning resources and that are much required to the good merchant. As we said at the beginning: otherwise, if

¹Presented at the São Paulo State Accounting Professionals Convention in 2019.

he is not a good counter in his business, he will walk like a blind, and many losses will occur..." (Hendriksen, 1999). Thus, the historical record of value was born.

With the Industrial Revolution such idea of procedure was taken to record books so the processes became more complex considering the increase of volumes and the necessity of control with the happened price oscillations. All in agreement with a foundation: the existence. If there exists, the document equivalent to its "input" so one can say that such merchandise existed, or such wage was used. The gain is recognized when occurs its output, what is also based in the rule of equivalent existence in certain document, that is the result is derivative and recognized by the fluctuations of inputs and outputs of this "energy" in this system. By evidence losses are also recognized when the results are established in respect to value and volume.

The practice of this valuation is based in the continuity principle and so one infers that the called provided benefit arises in the verification of this value and quantities fluctuation and the required expenses to revenue existence. Now the fluctuation is recognized by the existence of the document and the tied commitment to time line.

With the IFRS the idea of fair value became a guiding in the numbers of the accounting balance, that is, values may fluctuate in nonlinear way. The elected bases according to IASB include:

- (a) historic costs - assets are accounted for paid values, in cash or cash equivalents or for the fair value to acquire them, what is delivered at the time of acquisition. The liabilities are accounted for the values of what was received in exchange to obligation, and sometimes for values in cash or equivalents required to satisfy the liabilities;
- (b) current costs - assets are accounted for values in cash or equivalent, they must be paid if they or equivalent assets are acquired at present. The liabilities are accounted for values do not discounted, in cash or equivalent, required to settlement of the obligation at present;
- (c) realizable value (settlement value) - assets are held at values, in cash or equivalent, that would be achieved by selling the assets at a usual sale at present value. The liabilities are held at their settlement values, that is, the amounts, do not discounted, in cash or equivalent, expected to have to pay to satisfy the liability in normal operations;
- (d) present value - assets are held at current value, discounted the future flow of cash net inputs expected to be generated by the item in normal operations. The liabilities are held at current value, discounted the future flow of cash net outputs expected to be necessary to the settlement of liabilities in normal operations.

Thus, the numbers of a dynamic accounting are better represented to result measurement and making decisions in timeline (see Graphics 1 and 2). We propose a methodology based in graph studies in which the representation can capture the value oscillations in accounting balance through a topology. To assess this topology in three dimensions we use Kamada-Kawai algorithm.

II. THE COMPANY UNDER THE ACTION OF A "FORCES FIELD" - MARKETS AND ECONOMIC ADVENT

In a market economy the role of the companies is the production of goods and services. When a company invests it uses scarce resources to the production of goods and services to be sold to consumers. The present net value provides a notion of the value of the production over its costs that is the new created value. In case of an investment with present net value positive the company is using scarce resources to produce goods and services whose present value is higher than its production cost. There is creation of value, translated as expected future benefit to the company in timeline.

Information about the assets and liabilities of the society that will be evaluated may also not generate expected future benefits as well as the future sacrifices of the assets may not be observed in its expected cash flows. The usual literature brings wide conceptions of calculations about these hypotheses.

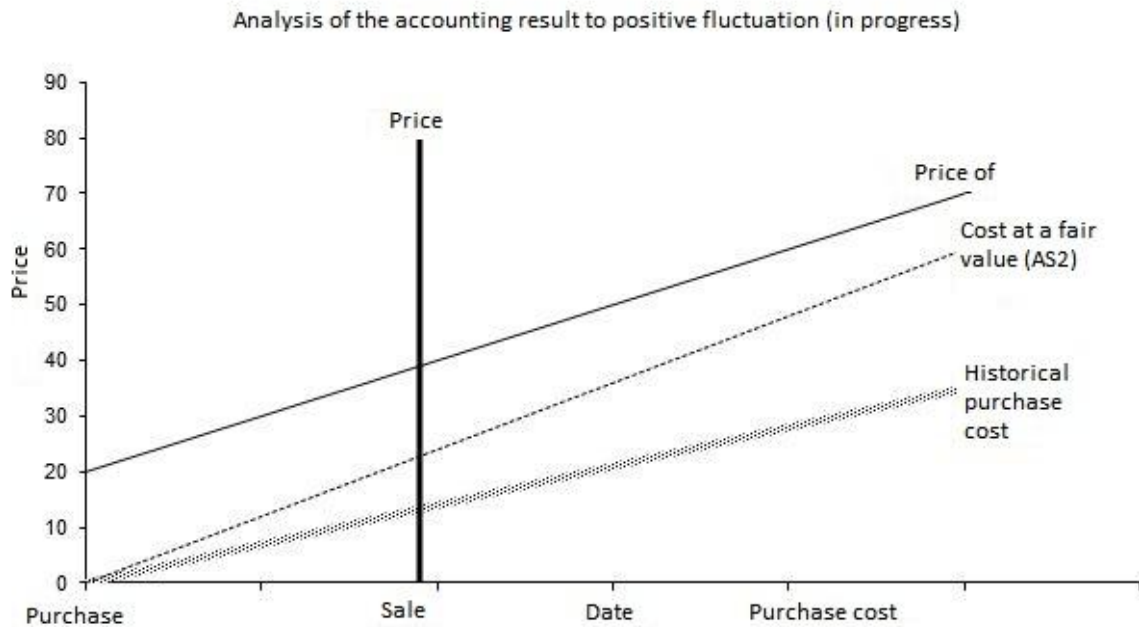
According to Edith Penrose (2002) in "The growth of the firm" (Penrose, 2002), the managers wish maximize profits in long term from the investments of the company. There seems to be a paradox between growing in successful way through reinvestment of profits and repay the owners relative to the capital they have applied taking risks. It happens that growing in a successful way may not be occurring in synchronization with market forces, thus we can infer that it is necessary endogenously organize these forces of the company with other pre-existing (exogenous).

Yet according to Penrose (2006) the company is simultaneously an administrative organization and a set of productive resources. The company links its resources with those of thirds such as goods and services organizing itself in an entity or "energy aggregate" or converting these energies in favourable entities to yourself or thirds.

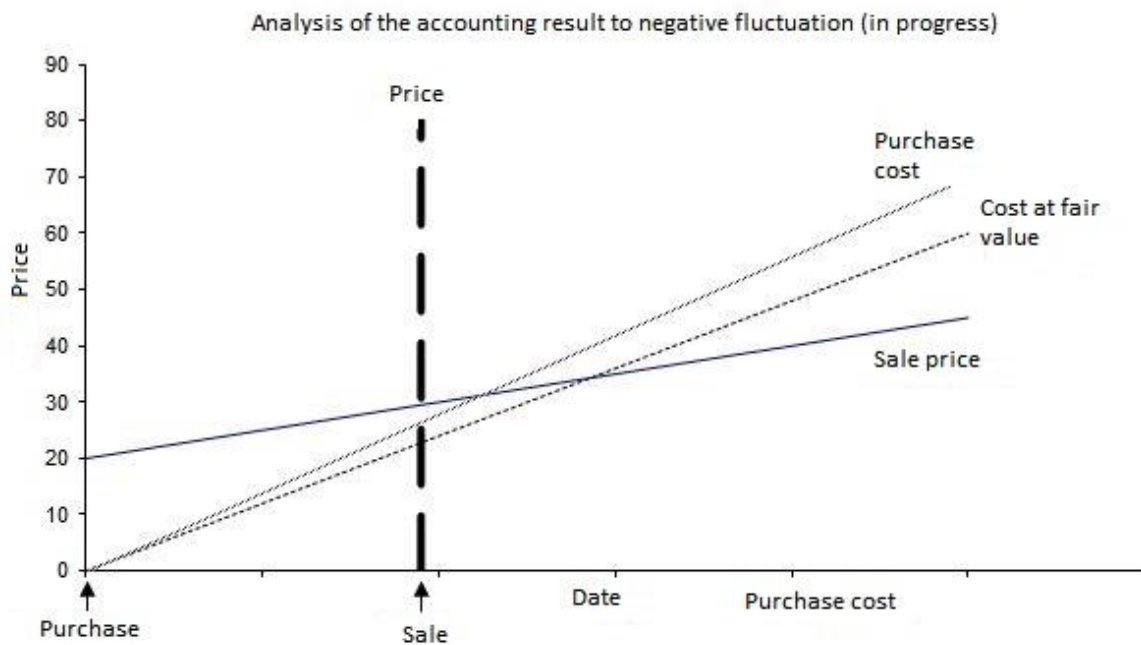
The administrative structure of the company is a creation of people in a rational way in which tasks are divided. On the other hand productive activities, according to Penrose (2006), are opportunities sought by shareholders or investors. We also must suppose that the companies do not grow and that failures follow in

waves what implies that the companies follow the others towards the end. However, some of companies will survive for long periods.

Graphic 1: Adapted from (Senéterre, 1980)



Graphic 2: Adapted from (Senéterre, 1980)



The administrative competence does not always is together with business competence. Administrative competence can be understood as the skills that the main managers have to mobilize resources, both financial or from other nature as labour etc. The called "business acumen" that can be present or not is the business opportunity vision hold by managers. From business competence we can highlight the company capacity in utilizing the energy until the maximum. As energy we define the utilization of resources and its cycles as purchase, sale, production etc.

The business and financial risks walk together (Robichek; Myers, 1965) and it is up to the manager minimize them through budget control, margins etc. It happens that not always is enough "to keep the key in the safe box" or project the same safe box to an uncertain future and sometimes remote.

We propose to study internal forces of the accounting with the assumption that accounting accounts are forces that interact each other in organized systems. We propose to see these forces in a dynamic form, as a film, and not in static form as in a newspaper. We use the computational algorithm called Kamada-Kawai (3). This algorithm (see Algorithm 1) starts from the study of graphs to facilitate the description of complex situations and allow a better visualization through a layout that represents systems in large scale in a better way.

As the ideas of Bourdieu, we can think a field as an abstract set of a multidimensional social space where agents exchange capital relations that when interact each other seek to increase or maintain their positions and intensities, with potential effects organized in structures that control part of the field or distort it according to its movements.

"More specifically, it is the agents, that is, the companies defined by the capital volume and structure that they hold which determine the field structure and the state of forces over the set (commonly called "sector" or "branch") of companies committed in the production of similar goods.

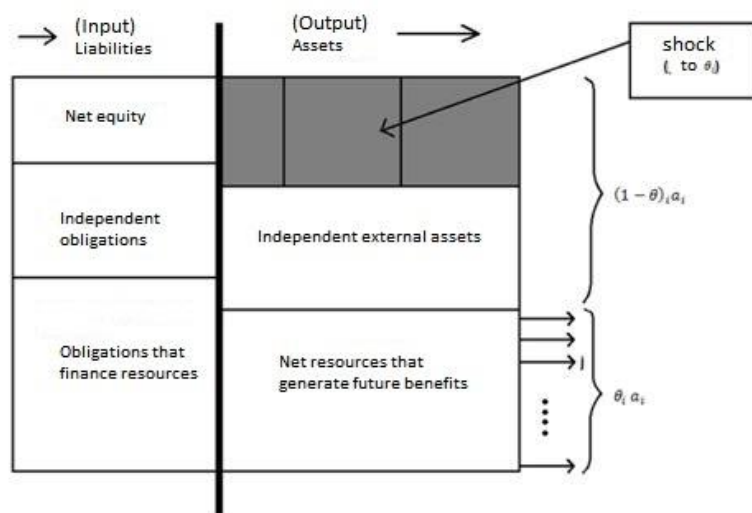
The companies exercise potential effects that vary in intensity, law of decrease and direction. They control a part of the field (market share) proportional to the size of its capital. The consumers, their behaviour would reduce to the field effect if they did not have a certain interaction with it (in function of the inertia, minimum). The associated weight to an agent depends on all the other points and on the relations among all the other points, that is, on all the understood space with a relational constellation." (Bourdieu, 2005).

A company is submerged in extrinsic factor fields that go beyond the limits of its control. as quoted by Penrose, 2006:

"After a discussion of the characteristics of a commercial company, its functions and the factors that influence its behaviour, we will go back to an examination of the forces inherent to companies nature that simultaneously create possibilities, provide inducing factors and limit the expansion rate that they can undertake, or plan and undertake in a given time interval." (Penrose, 2006)

But what forces are able to affect a business plan of the company such that they do not become predictable? First we remember that a company is part of a big and complex network subjected to shock waves that propagate in economic cycles. We can quote a borrowing company to reinvestment. As example of shock wave propagated in network, if the interests increase there will be a big possibility that the investments will reduce in the assets and so the liabilities connected to this company also will be affected (Penrose, 2006).

Figure 3: Impact waves on an accounting balance represented by the shaded region. The factor a is the multiplicity in which the asset is strengthened. Changed by Robert May - Santa Fe Institute.



On the left side of Figure 3 there is the resource input (origin) that will finance the right side (assets or resources application). However, the above system is not isolated. There are many connections in both side right and left of system.

To a better understanding it is needed to see that the company is inserted in a complex economic system inside a complex network. Thus, the circulating capital in the economy passes through the companies, the consumers and the banks.

According to (Boadway; Pestieau, 2003) the interests have a central role in the impact of debts because they are directly correlated, as they fluctuate with the existing risks. Such interests are distributed on risks components in network. According to their prescription we have the following proposition:

"A meaningful increase at risk decreases the borrowing activities and implies a decrease in the equivalence certainty of the interest rate to a neutral risk lender, although real interest rates probably increase."

The risk increases according to the realized loan volume. (Boadway; Pestieau, 2003) assures that if the bank keeps its interest rate fixed means a form of reduction because to the borrower without the increase of the rate this loan will be more interesting.

On the other hand, under such conditions, the lender would consider the risk neutral and the loan would become more attractive. When occurs an increase in the interest rate, there will be an increase at the risk, showing that credit conditions are no longer stables. On the opposite side, the one of the investor, we have the contrary because it is interesting the increase of this rate. Continuing (Boadway; Pestieau, 2003) states that:

"...what matters is if the increase on the interest rate is more than necessary to compensate the greater probability of default. We focus in the expected return and we call this the equivalence certainty of interest rate to a neutral risk lender. In other words, we question what happens with the interest rates fitted to neutral risk. ...This is a complicated question because the probability of default is subjective and may differ between the borrower and the lender. ... the borrower can believe that the interest rate is more than enough to cover the chances of default (and specially at the margin, if the lender charge interest rates higher and higher each time the borrower take more loans), while the lender believes that does not cover even the risk premium."

Below the author presents a formulation that demonstrates that interests has a projective movement dynamic. This interest depends on two components, the one of business cycle of the borrower, and intrinsically of his own margins. This means that a discontinuity of the capital in the business cycle of this borrower will default him, affecting the capital primary dealer. Thus, the modelling will occur as presented. If the safe interest rate was p and the risk of banks was neutral, the interest rate fitted to the risk r_a is found through the solution of the following system of two equations:

$$p = r_a (1 - P_b) + \int_{\theta_f}^{\infty} \left(\frac{Y_f - c_f}{B} - 1 \right) dF(\theta) \quad (1)$$

where B represents the amount that bank lends to a borrower, for example for a company. Here c_f is the bankruptcy cost and Y_f is the return in the project implemented by the borrower company which depends on the state of business cycle with its partners θ , where θ_f is the value at which the company enters the bankruptcy process. Factor P_b is the probability of bankruptcy of this company:

$$P_b = \text{Prob} \{ Y_f(\cdot, \theta) \leq (1 + r_a) B \} \quad (2)$$

The risk premium charged by a bank averse to risk (or to market) is the excess of interest rate charged r over r_a . Note that to this company the charged interest rate seems much greater than the one received by the bank since it loses all the outputs Y_f in case of bankruptcy, whereas the bank can extract only the amount P_b of the bankrupt assets.

The bank can cover exactly the equivalence of interest rate to a neutral risk lender r_a (as defined below). So the subjective interest rate to the company (expected net cost of the loan) r_f clearly is greater than r_a (expected net return by the bank):

$$r_f = r_a (1 + P_b) + \int_{\theta_t}^{\infty} \left(\frac{Y_f}{B} - 1 \right) dF(\theta) \quad (3)$$

Thus to the bank with a diversified portfolio what matters it is not the default probability of this loan but if there will be avalanche of defaults such that this probability increases. If the bank charges only the interest rate that covers the increased risk, then the return remains unchanged but the variability of its return increases - there is a greater chance when a big portion of loans be impacted by this set of default.

III. ACCOUNTING SYSTEM

According to Horngren (Horngren; Sundem; Stratton, 2004) an accounting system is a formal way of gathering data that will allow the coordination on taking collective decisions as well as helping in this process in order to achieve goals and objectives established by the organization. As it is known, accounting systems are made by reports with structured accounts such that the accounting logic is structured in an organic way. Such data as classified in different ways and are grouped through the record in groups of accounts with an ordered pair which consists of two elements (α, β) , where one of them is designated, say α , as the first element and the other, say β , as the second element.

This concept allows interpreting the assets as properties, liabilities as debts and the expense accounts and resources inputs as changes in net capital. In the evolutionary process economic and financial transactions are recorded by the same ordered pair, that is, the meaning of transactions is related to society activities. The summation of transactions represents an accumulated amount that influences the movement and the characterization of the company. We suppose that the accounting links among accounts are links of a graph which in turn is a network with representation $G = (V, A)$, where the nodes or vertex, N , form the set N and the edges, connections or links, M , form the set A and identify the relations among the elements of the system.

IV. STRUCTURE OF A GRAPH

In formal definitions of (Bondy; Murty et al., 1976) graph structures are defined as

$$G = (V(G), E(G), \psi_G) \quad (4)$$

where

$$V(G) = \{v_1, v_2, v_3, v_4, v_5\} \quad (5)$$

$$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\} \quad (6)$$

and ψ_G is defined as

$$\begin{aligned} \psi_G(e_1) &= v_1 v_2, \\ \psi_G(e_2) &= v_2 v_3, \\ \psi_G(e_3) &= v_3 v_3, \\ \psi_G(e_4) &= v_3 v_4 \end{aligned}$$

$$\begin{aligned} \psi_G(e_5) &= v_2 v_4, \\ \psi_G(e_6) &= v_4 v_5, \\ \psi_G(e_7) &= v_2 v_5, \\ \psi_G(e_8) &= v_2 v_6 \end{aligned}$$

and other example is given by

$$H = (V(H), E(H), \psi_H) \quad (7)$$

where

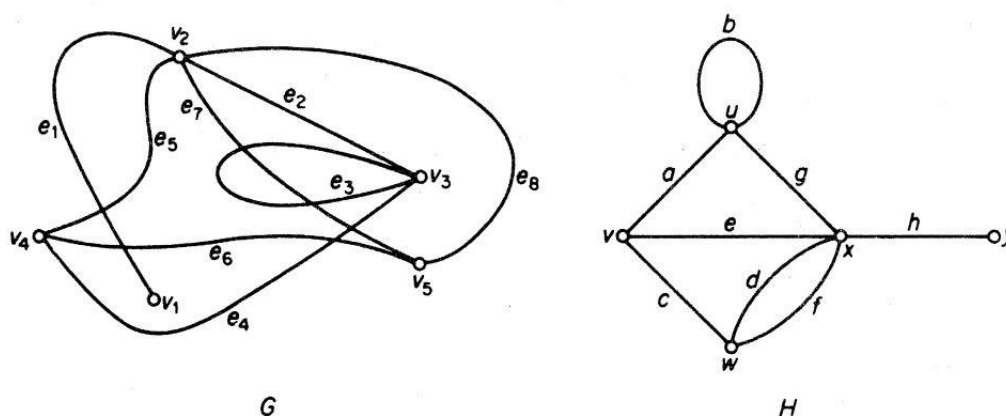
$$V(H) = \{u, v, w, x, y\}$$

$$E(H) = \{a, b, c, d, e, f, g, h\}$$

and ψ_H is defined as

$$\begin{aligned} \psi_{H(a)} &= uv \\ \psi_{H(b)} &= uu, \\ \psi_{H(c)} &= vw \\ \psi_{H(d)} &= wx \\ \psi_{H(e)} &= vx \\ \psi_{H(f)} &= wx \\ \psi_{H(g)} &= ux \\ \psi_{H(h)} &= xy \end{aligned}$$

Figure 4: Diagram of graphs G and H (Bondy; Murty et al., 1976).



The use of graphs to represent accounting movements are very relevant because demonstrate the different paths and links of accounting accounts. According to Bondy and Murty:

"Graphs have this name because they can be represented graphically and is this graphic representation that help us to understand many of its properties. Each vertex is indicated by a point and each edge is indicated by a line connecting the points that represent its extremity. There is no a unique way to design a graph; the relative positions of points that represent vertex and the lines that represent edges have no meaning." (Bondy; Murty et al., 1976)

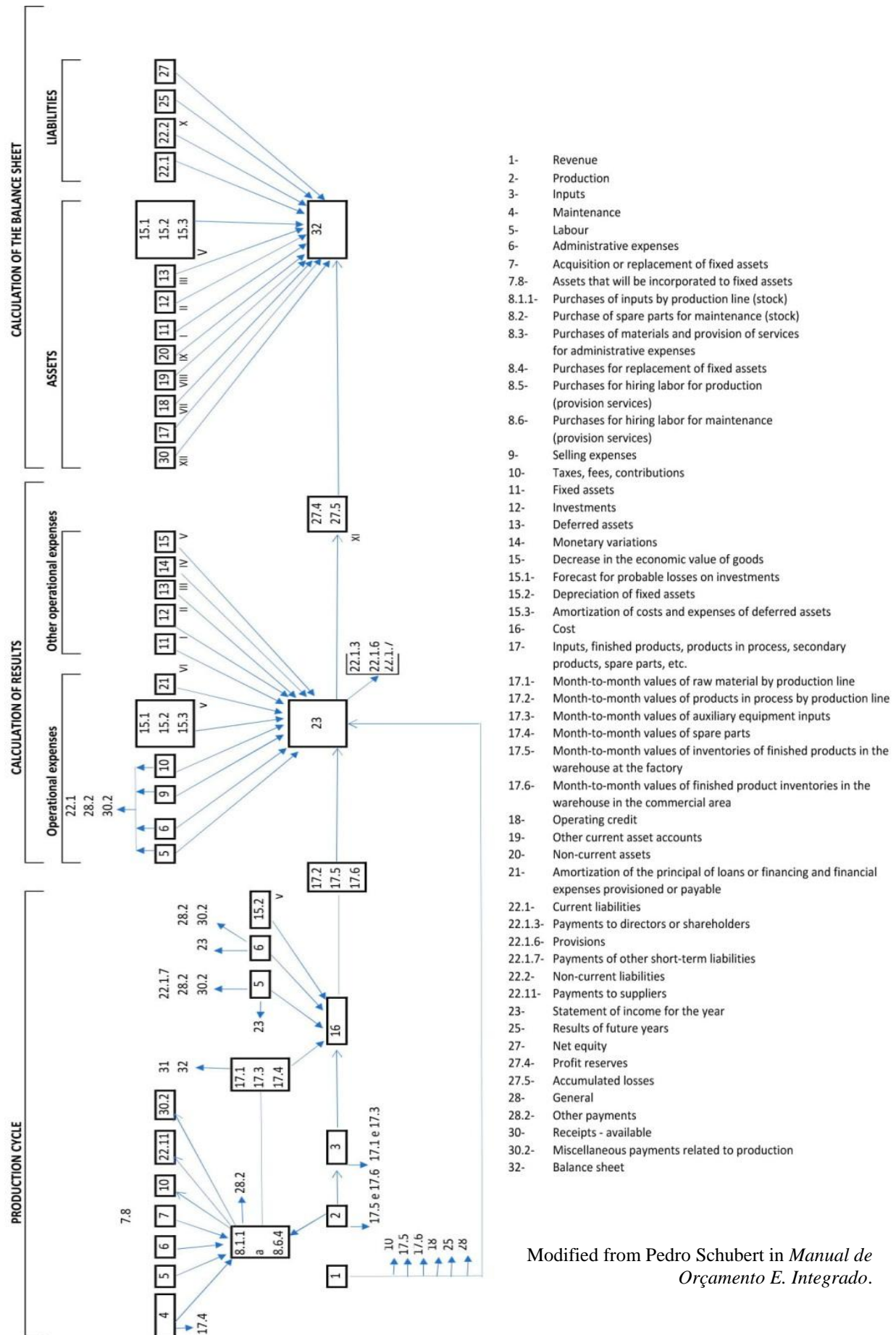
At first sight the idea of graph supposes mere connections of accounts that seems to have different properties to measurement purposes and shows that function in a different way when we want to connect an account to the other through the values of the ordered pair (a,b). However, when we assign values to vertices everything changes and now the weights have influence in the connections of the accounts.

The measurements in accounting presuppose to assign numeric values to objects, events, facts and acts related to the company. This measurement occurs through a classification, a method and a certain value judgment that are represented in the connections and that from now we intend represent in form of graphs. All this effort to expose such facts in a balance built in a logic way.

A possible accounting representation of this accounts flow is the representation of Figure 5, where the links among the groups of accounting accounts are according to accounting rules and principles. As we can see the links are simple representations of possible connections of the accounting accounts that connect each other as defined in (4).

With current computational resources it is possible to demonstrate both the static position (balance) in a timeline as to show how was formed the final position during the process, in independent and intermediate states. Considering possible regimes or of competence or of cash we have that the chronology is preserved and we can assure that the states will be preserved at a certain Δt .

The analogy we suppose is that the connections (ordered pair) are movements of forces whose topology is different to each company. The movements among these accounts sum an amount of values and according to the amount of these connections they will determine the size of the edge defined in (6).



Modified from Pedro Schubert in *Manual de Orçamento E. Integrado.*

The origins and resources application at each company have different components and depends on the manager performance in mobilizing such components.

Thus with the knowledge of the movement of the topology it is possible to predict and reveal a direction of the stationary states that must contribute both in the knowledge of the company potential as in the monetary decisions and of management such as the uncertain factors of a purchase, a sale, production etc.

With these assumptions we need to measure how these forces are disposed and how would result in a topology in which we postulate that represent such potential and kinetic "energies". For this we have the algorithm below.

V. KAMADA-KAWAI ALGORITHM

In this paper we try to understand the forces interacting internally through accounting movements. Kamada-Kawai's algorithm (Kamada-Kawai et al. 1989) uses the graph structure in space and simultaneously the elastic potential energy theory through a spring system. Then in the algorithm we can change the connections of springs to movements or accounting amounts of the accounts in a trial balance.

The proposal of Kamada-Kawai is the representation of the potential energies of the connections among nodes through the springs where the total energy to all the springs (connections among nodes) is given by

$$E_{\text{tot}} = \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{1}{2} k_{i,j} \left((x_i - x_j)^2 + (y_i - y_j)^2 - 2l_{i,j} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} + (l_{i,j})^2 \right) \quad (8)$$

The proposal of the algorithm is to find the coordinates of points represented by the nodes of graph in a minimum space among these points. So it proposes to find the global minimum of the energy of the connections among these points and the minimization condition is

$$\frac{\partial E_{\text{tot}}}{\partial x_m} = \frac{\partial E_{\text{tot}}}{\partial y_m} = 0 \quad \text{para } 1 \leq m \leq n, \quad (9)$$

and the partial derivatives are

$$\frac{\partial E_{\text{tot}}}{\partial x_m} = \sum_{m \neq j=1}^n k_{m,j} \left\{ (x_m - x_j) - \frac{l_{m,j} (x_m - x_j)}{\left[(x_m - x_j)^2 + (y_m - y_j)^2 \right]^{1/2}} \right\} \quad (10)$$

$$\frac{\partial E_{\text{tot}}}{\partial y_m} = \sum_{m \neq j=1}^n k_{m,j} \left\{ (y_m - y_j) - \frac{l_{m,j} (y_m - y_j)}{\left[(x_m - x_j)^2 + (y_m - y_j)^2 \right]^{1/2}} \right\}. \quad (11)$$

Solving equations (9) and (10) by Newton-Raphson method for two dimensions we get a local minimum that satisfies the iteration with a step given by

$$\Delta_m = \sqrt{\left(\frac{\partial E_{\text{tot}}}{\partial x_m} \right)^2 + \left(\frac{\partial E_{\text{tot}}}{\partial y_m} \right)^2} \quad (12)$$

The springs have a position displacement given by δ_x and δ_y , and the final positions were defined as

$$x_m^{(t+1)} = x_m^{(t)} + \delta x, \quad y_m^{(t+1)} = y_m^{(t)} + \delta y \quad \text{for } t = 0, 1, 2, \dots \quad (13)$$

The Jacobian for x_m and y_m can be defined as

$$\frac{\partial^2 E}{\partial x_m^2} \left(x_m^{(t)}, y_m^{(t)} \right) \delta x + \frac{\partial^2 E}{\partial x_m \partial y_m} \left(x_m^{(t)}, y_m^{(t)} \right) \delta y = - \frac{\partial E}{\partial x_m} \left(x_m^{(t)}, y_m^{(t)} \right) \quad (14)$$

$$\frac{\partial^2 E}{\partial y_m \partial x_m} \left(x_m^{(t)}, y_m^{(t)} \right) \delta x + \frac{\partial^2 E}{\partial y_m^2} \left(x_m^{(t)}, y_m^{(t)} \right) \delta y = - \frac{\partial E}{\partial y_m} \left(x_m^{(t)}, y_m^{(t)} \right) \quad (15)$$

Finally the coefficients of partial derivatives are

$$\frac{\partial^2 E}{\partial x_m^2} = \sum_{i \neq m} k_{mi} \left\{ 1 - \frac{l_{mi} (y_m - y_i)^2}{\{(x_m - x_i)^2 + (y_m - y_i)^2\}^{3/2}} \right\} \quad (16)$$

$$\frac{\partial^2 E}{\partial x_m \partial y_m} = \sum_{i \neq m} k_{mi} \frac{l_{mi} (x_m - x_i) (y_m - y_i)}{\{(x_m - x_i)^2 + (y_m - y_i)^2\}^{3/2}} \quad (17)$$

$$\frac{\partial^2 E}{\partial y_m \partial x_m} = \sum_{i \neq m} k_{mi} \frac{l_{mi} (x_m - x_i) (y_m - y_i)}{\{(x_m - x_i)^2 + (y_m - y_i)^2\}^{3/2}} \quad (18)$$

$$\frac{\partial^2 E}{\partial y_m^2} = \sum_{i \neq m} k_{mi} \left\{ 1 - \frac{l_{mi} (x_m - x_i)^2}{\{(x_m - x_i)^2 + (y_m - y_i)^2\}^{3/2}} \right\} \quad (19)$$

Now the values of δ_x and δ_y , can be calculated from equations (13)-(18).

VI. RESULTS

Using a test company and data that generated trial balance we get the following geometric expositions or graphs:

Figure 4: To a test company. In the left side (a) we show the trial balance relative to March, while in the right side (b) we show the trial balance relative to June.

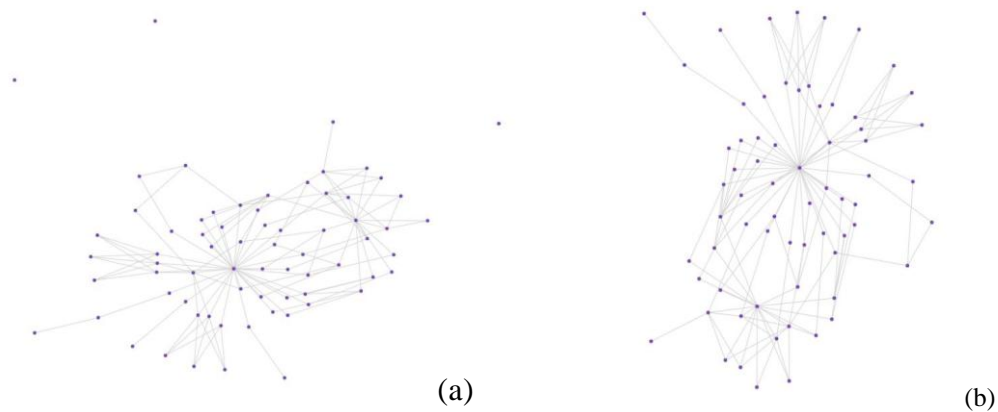
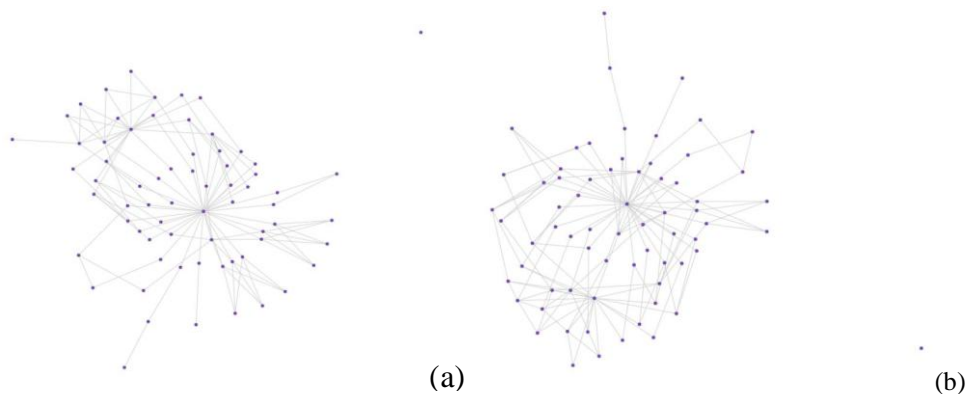


Figure 5: To a test company. In the left side (a), we show the trial balance relative to September, while in the right side (b) we show the trial balance relative to December.

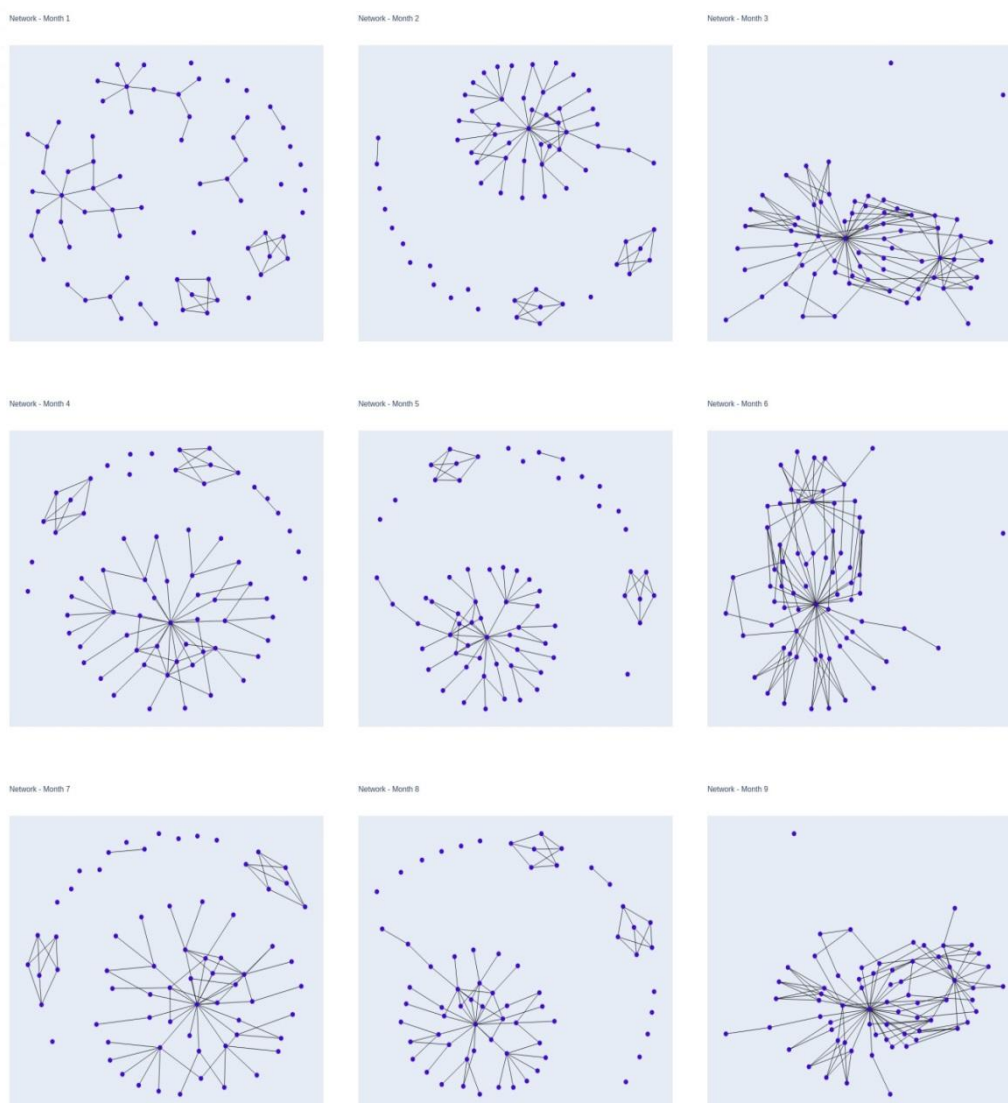


As previously mentioned, the accounts represent links of "energies" that compose a topology of the company. If the company is submerged in a "force field" representing external components so there is an internal movement in such companies.

It is possible to identify the concentration of these forces and its centre in a complex network. Also is possible to identify which accounts mobilized the "energetic" movement internally generated.

The main contribution of this model is the possibility to make predictions from the stationary states that compose the wealth of entities.

It is possible to determine month by month the evolutionary processes designed in the topology and to achieve a certain behaviour standard that would be used in economic analysis and forecast of the company (Figure 6).



VII. CONCLUSION

In this paper we present an approach to view a trial balance through links of accounting accounts disposed in a network using the algorithm Kamada-Kawai. This algorithm is stabilized by a "minimum energy" that allows a topology in form of graphs that is pretty consistent and represent a set of data in a better way. It is possible to associate the ordered pair (known as the debit and credit accounts) to a graph definition: $G = (V,E)$, where V is the set of vertices (integers) and E is the set of edges. To each edge is assigned the amount of the accounts. The central idea is to create a topology where is possible to identify the concentration and disposition of local flows (movements of the accounts). Local flows can be represented and stored in a visible data structure that can translate information and conclusion with respect to the accounts, the study object. It is possible to identify visually certain data clusters that can take to an analysis of present movements. The study of the graph

